Superradiance and Cascaded Decay

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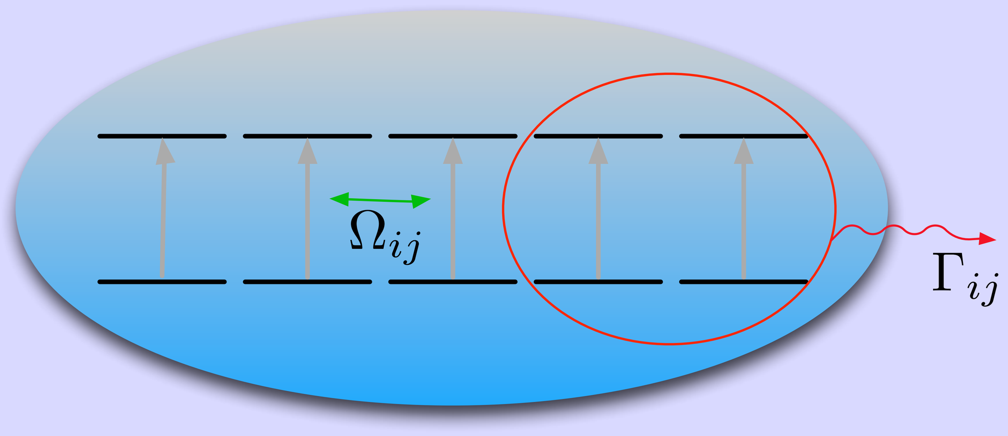
1 Introduction and Goals

- Study of collective decay processes of atoms in optical lattices
- Analysis of collective effects in different geometric configurations by analytical and numerical methods
- Going beyond the single-excitation or small sample limit and treating the full system for arbitrary configurations
- Predicting, understanding and controlling energy shifts and linewidths of atomic ensembles in optical lattices
- Contributing to precision gains in Optical Lattice Clocks
- *Next: Study of these ensembles inside optical cavities and contributing to the advancement of narrow-linewith lasers*

2 Model

N Two-Level systems coherently coupled through resonant dipole-dipole interactions mediated by virtual photons



The system's Hamiltonian is given by

$$\hat{H} = \sum_i \omega_i \hat{S}_i^+ \hat{S}_i^- + \sum_{i \neq j} \Omega_{ij} \hat{S}_i^+ \hat{S}_j^-$$

where ω_i is the transition energy in the i-th atom and Ω_{ij} the resonant coupling between atom i and j. The dissipation is treated in an ensemble average approach with the master equation

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}\left[\hat{H}, \hat{\rho}\right] - \mathscr{L}_{cd}\left[\hat{\rho}\right]$$

with the Liouvillian

$$\mathscr{L}_{cd}\left[\hat{\rho}\right] = \frac{1}{2}\sum_{ij} \Gamma_{ij}\left(\hat{S}_i^+ \hat{S}_j^- \hat{\rho} + \hat{\rho} \hat{S}_i^+ \hat{S}_j^- - 2\hat{S}_i^- \hat{\rho} \hat{S}_j^+\right)$$

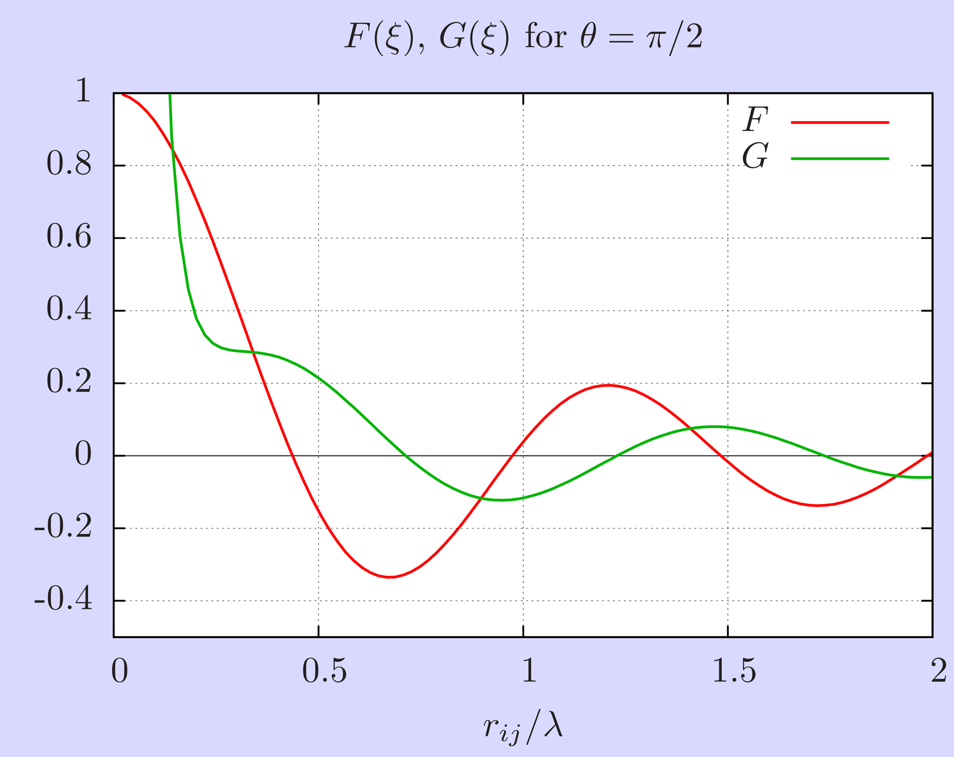
featuring the collective damping Γ_{ij} between the i-th and j-th atom. The collective coupling and decay depend on the system's geometry and the orientation of the transition dipole relative tot the atom's spacial distribution.

$$\Omega_{ij} = \frac{3\sqrt{\Gamma_i \Gamma_j}}{4} G(k_0 r_{ij}) \qquad \Gamma_{ij} = \frac{3\sqrt{\Gamma_i \Gamma_j}}{2} F(k_0 r_{ij})$$

For $\xi = k_0 r_{ij} = 2\pi r_i j/\lambda_0$ and the polarisation angle θ we have

$$F(\xi) = \left(1 - \cos^2\theta\right)\frac{\sin\xi}{\xi} + \left(1 - 3\cos^2\theta\right)\left(\frac{\cos\xi}{\xi^2} - \frac{\sin\xi}{\xi^3}\right)$$

$$G(\xi) = -\left(1 - \cos^2\theta\right)\frac{\cos\xi}{\xi} + \left(1 - 3\cos^2\theta\right)\left(\frac{\sin\xi}{\xi^2} + \frac{\cos\xi}{\xi^3}\right)$$

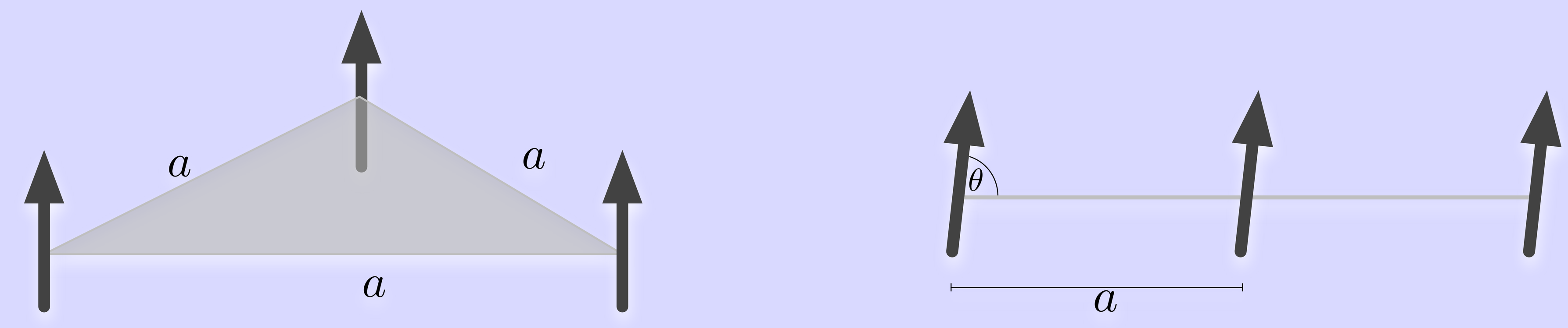


A distance of special interest is the magic wavelength for the clock transition in ^{87}Sr (trapping wavelength at which light shifts are cancelled)

$$\frac{\lambda_m}{2\lambda_0} = 0.5824$$

3 Configurations

Systems considered include an equilateral triangle of length a with polarisation angle $\theta = \pi/2$ and a chain of three atoms with lattice constant a and arbitrary polarisation angle. In numerical simulations we go to longer chains of up to seven atoms



Approach: the triangle can be treated fully analytically, the chain of three already needs numerics, but is still explicitly solvable. Longer chains are best done in MCWF-simulations.

Ramsey Signal

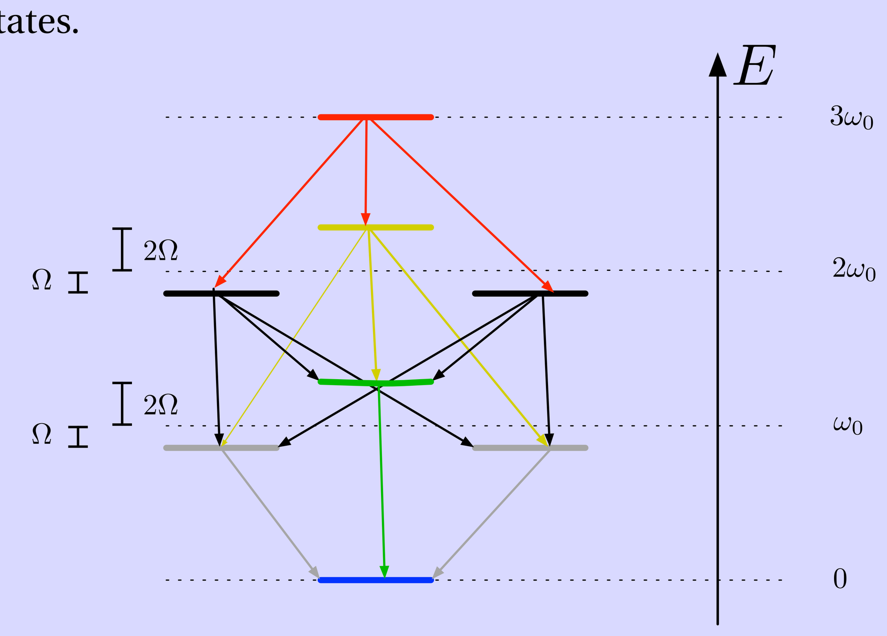
With these two configurations we look at the Ramsey signal, that emerges if we start with all atoms in the ground state, apply a resonant $\pi/2$-pulse ('Hadamard'-gate) to each atom, then leave the systems to its free dynamics, and after a time t apply a second (in-phase) $\pi/2$-pulse, again to each atom. The plot shows the survival probability of the totally excited state e as a function of the time t in between the two pulses.

$$|g\rangle^{\otimes N} \xrightarrow{\frac{\pi}{2}-\text{Pulse}} (|g\rangle + |e\rangle)^{\otimes N} \longrightarrow U(t) \xrightarrow{\frac{\pi}{2}-\text{Pulse}} |f\rangle \longrightarrow P_{|e\rangle^{\otimes N}}$$

4 Results

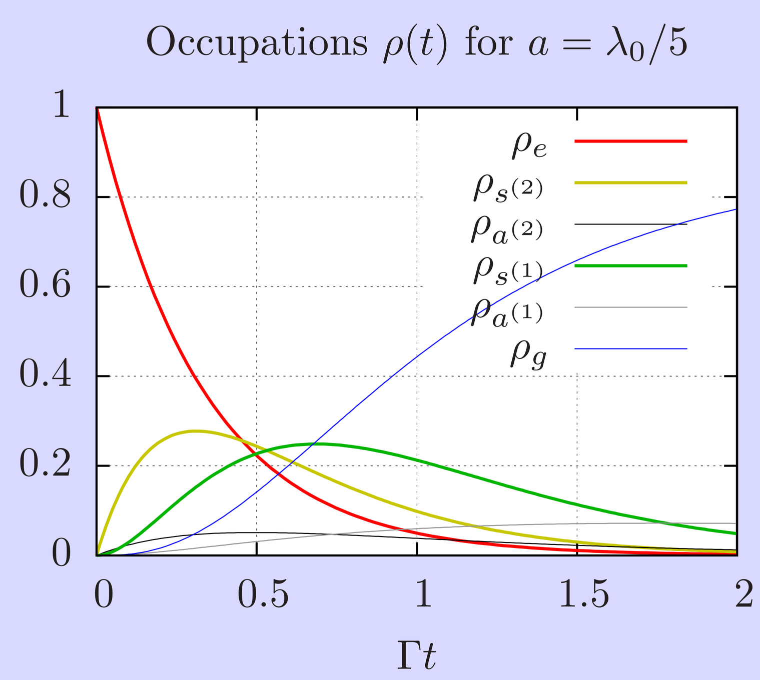
Triangle

The decay cascade of the triangle shows a twofold degeneracy for the single- and bi-excitation downwards-shifted states.



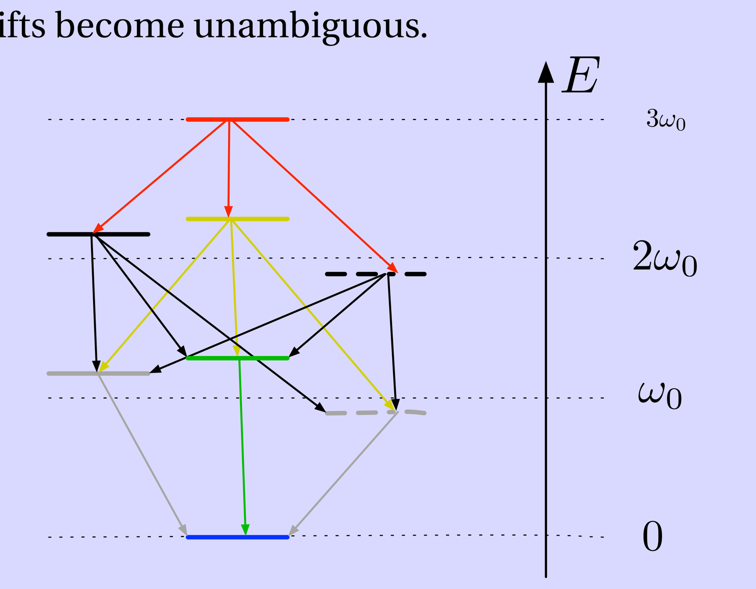
Intermediate occupations are of the form

$$\rho_{\text{interm}}(t) = A\underbrace{\left(1 - \exp\left(-\gamma_f t\right)\right)}_{\text{Feeding}}\underbrace{\exp\left(-\gamma_d t\right)}_{\text{Decay}}$$

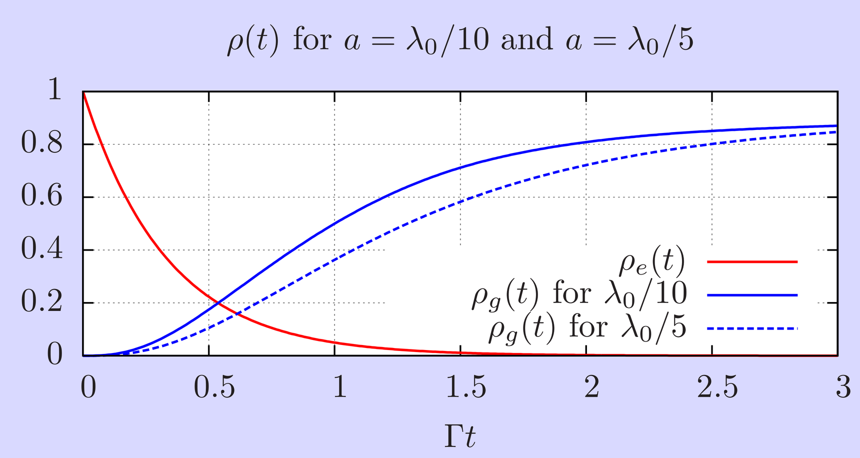


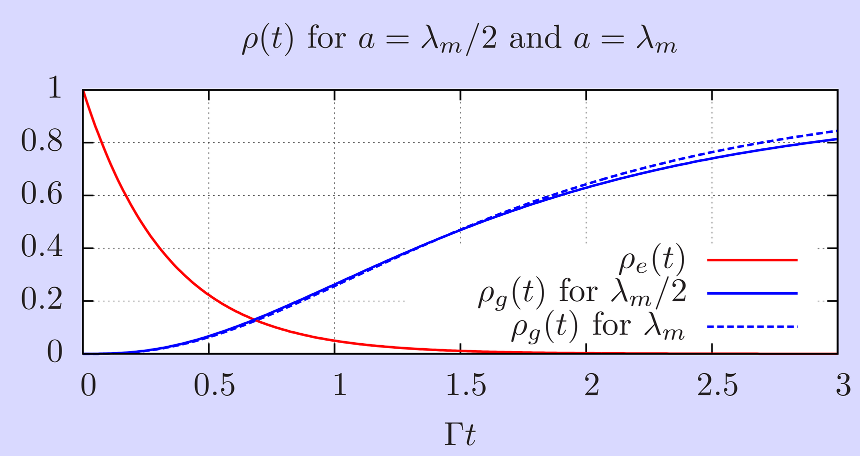
Chain

By less symmetry in the chain the degeneracy is lifted and the shifts become unambiguous.



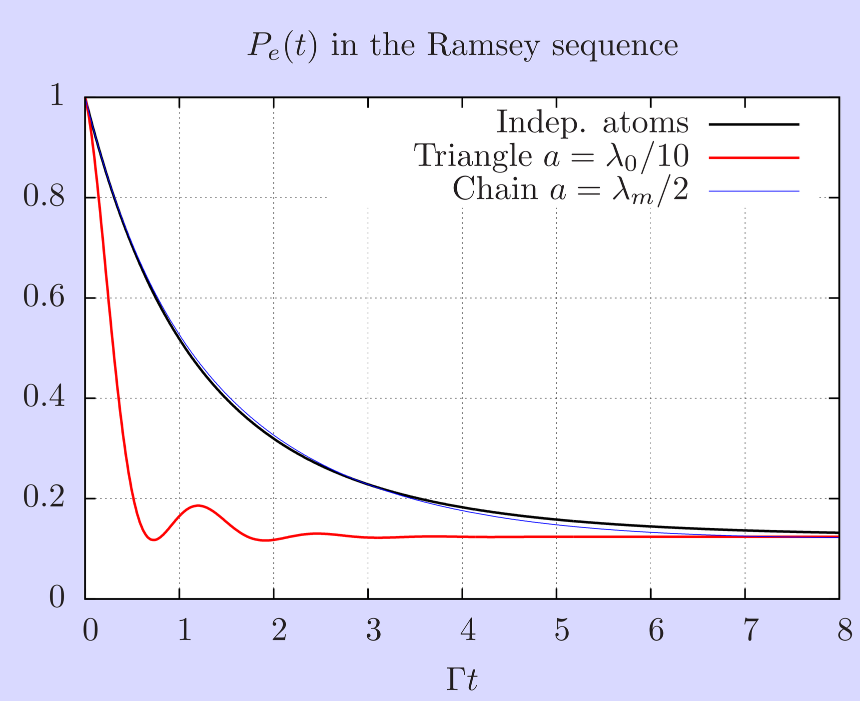
The closer the atoms are positioned, the faster the decay process will populate the ground state.





Ramsey Signal

We now look at the survival probability of the fully inverted state of these configurations in the Ramsey sequence. The signal is compared to the case of three independent atoms. Notice, that in any configuration $P_e(t \to \infty) = 1/8$.

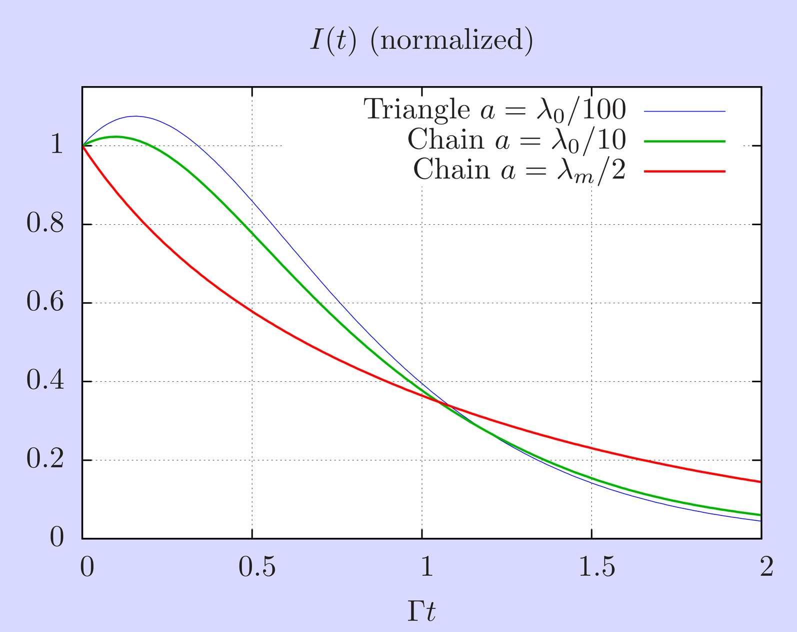


Rate of Energy Emission

To quantify how fast the system's energy is lost to the environment we define

$$I(t) = -\partial_t \langle \hat{H} \rangle_{\rho(t)}$$

as the rate of energy emission. The plot shows the emission rate for different distances. Notice that the distances need to be quite small in order to show a significant superradiant behaviour.



References and Acknowledgements

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