Collective Decay Cascades

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1 Concept

We consider N two-level atoms in an optical lattice with dipole-dipole interaction described by the Master equation

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \frac{1}{2}\sum_{i,j}\Gamma_{ij}\left(S_i^+ S_j^- \rho + \rho S_i^+ S_j^- - 2S_i^- \rho S_j^+\right)$$

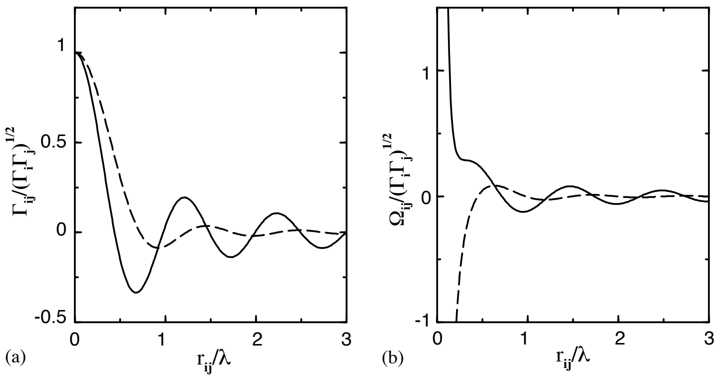
with the Hamiltonian

$$H = \hbar\sum_i \omega_i S_i^z + \hbar\sum_{i\neq j}\Omega_{ij}S_i^+ S_j^- + H_L,$$

where $H_L = \frac{1}{2}\sum_i \left[\Omega(r_i)S_i^+ \exp\left(i(\omega_L t + \phi_L)\right) + \text{h.c.}\right]$ describes the interaction with some coherent driving field. The collective dipole-dipole shift and the collective spontaneous emission rates are a function of the single atom lifetime and the geometry and appear due to the mutual coupling of the atomic dipoles.

$$\Gamma_i = \frac{\omega_i^3 \mu_i^2}{3\pi\epsilon_0\hbar c^3} \qquad \Gamma_{ij} = \sqrt{\Gamma_i\Gamma_j}F(k_0 r_{ij}) \qquad \Omega_{ij} = \frac{3}{4}\sqrt{\Gamma_i\Gamma_j}G(k_0 r_{ij})$$

Disregarding the exact analytical expression the shapes of F and G are plotted below and it is noteworthy that they are a function of the resonant wavelength, the interatomic separation and the relative angle between the atomic transition dipole and distance between the atoms (plot: Ficek 2002). F is plotted on the left-hand side, G on the right-hand side with the solid lines showing the function for s$\mu \perp r_{ij}$ and the dotted line refers to a parallel orientation.



2 Example for two atoms

For two identical atoms the bare atomic states are

$$|g_1 g_2\rangle\,, |e_1 g_2\rangle\,, |g_1 e_2\rangle\,, |e_1 e_2\rangle$$

with the corresponding energies $E_{gg} = -\hbar\omega_0$, $E_{ge} = E_{eg} = 0$, $E_{ee} = \hbar\omega_0$. Now also taking into account the dipole-dipole interaction the Hamilton has the form

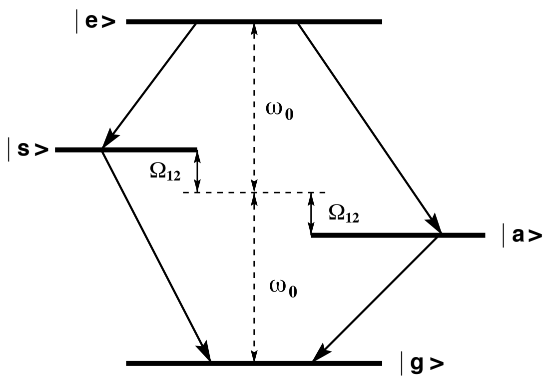
$$H = \frac{\hbar\omega_0}{2} \sum_{i=1}^{2} S_i^z + \sum_{i \neq j} \Omega_{ij} S_i^+ S_j^-\,,$$

which written in the bare atomic states looks like

$$H = \hbar \begin{pmatrix} -\omega_0 & 0 & 0 & 0 \\ 0 & 0 & \Omega & 0 \\ 0 & \Omega & 0 & 0 \\ 0 & 0 & 0 & \omega_0 \end{pmatrix}$$

and leads to new eigenstates

$$|e\rangle = |e_1 e_2\rangle$$
$$|s\rangle = \frac{1}{\sqrt{2}} \left(|e_1 g_2\rangle + |g_1 e_2\rangle\right)$$
$$|a\rangle = \frac{1}{\sqrt{2}} \left(|e_1 g_2\rangle - |g_1 e_2\rangle\right)$$
$$|g\rangle = |g_1 g_2\rangle$$



The corresponding energies are $E_e = \hbar\omega_0$, $E_s = \hbar\Omega$, $E_a = -\hbar\Omega$, $E_g = -\hbar\omega_0$. Employing the projection and correlation operators $|i\rangle\langle j|$ with $i, j \in \{e, s, a, g\}$ we rewrite the Master equation to

$$\dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \left(\frac{\partial \rho}{\partial t}\right)_s + \left(\frac{\partial \rho}{\partial t}\right)_a,$$

where

$$H = \hbar \left[\left(|e\rangle \langle e| - |g\rangle \langle g| \right) \omega_0 + \left(|s\rangle \langle s| - |a\rangle \langle a| \right) \Omega \right]$$

and $(\partial\rho/\partial t)_s$ and $(\partial\rho/\partial t)_a$ are the super- and subradiant decay chains with

$$\begin{aligned}
\left(\frac{\partial\rho}{\partial t}\right)_s =& -\frac{\Gamma + \Gamma_{12}}{2} \left[\left(|e\rangle \langle e| + |s\rangle \langle s| \right) \rho + \rho \left(|e\rangle \langle e| + |s\rangle \langle s| \right) \right. \\
& \left. -2 \left(|s\rangle \langle e| + |g\rangle \langle s| \right) \rho \left(|e\rangle \langle s| + |s\rangle \langle g| \right) \right] \\
\left(\frac{\partial\rho}{\partial t}\right)_a =& -\frac{\Gamma - \Gamma_{12}}{2} \left[\left(|e\rangle \langle e| + |a\rangle \langle a| \right) \rho + \rho \left(|e\rangle \langle e| + |a\rangle \langle a| \right) \right. \\
& \left. -2 \left(|a\rangle \langle e| - |g\rangle \langle a| \right) \rho \left(|e\rangle \langle a| - |a\rangle \langle g| \right) \right]
\end{aligned}$$

and we observe that the two decay chains decouple from each other.

3 Direction

The plan is now to generalise this formalism to N atoms and see how the decay chains are correlated with the eigenvalues and eigenstates of the matrix that can be built up from Γ_{ij}.

We already know that there can be very small ($\approx 10^{-4}$) and very large (see plots) collective spontaneous emission rates.